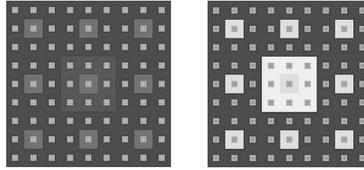


Mathematical Visualization series

Deconstructing Cantor set theory

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ABSTRACT. – In 2005, I developed a series of visual investigation on optical phenomena for an advanced program in Visual Communication. Most of the background and information material was based on cognitive processes, neurological and topological research from scientists of the turn of the century. Earlier this year, I became interested on the technical challenge of the Sierpinski’s fractal representation in the digital environment. Sierpinski also produced most of his work at the beginning of the 1900. This interesting coincidence guided me to attempt to join the two fields of inquiry and highlight the similarities and overall coherence of an intellectual and scientific trend at a particular time in history that helped map the premises of today’s visual communication vocabulary.

RESUME. - The following presentation endeavors to bring together elements of fractal topology, optical distortions and color theory in the digital environment and examine the effectiveness of integrating various methodologies in the production of a visually coherent proposition.

CONTENT

Introduction	2
I The Mathematical Grid.....	3
II Optical Illusion.....	9
III Color manipulation	11
Conclusion.....	12
<i>Acknowledgment & References.....</i>	13

INTRODUCTION

The purpose of this project is to explore the dynamic of a fractal phenomenon interacting with elements of optical illusion in an esthetic and visual communication framework.

The following experiment owes much to Laplacian mathematic, more precisely the Cantor set principle and the Sierpinski projection on a two dimensional plan also known as the Sierpinski carpet [1] [2].

An opportunity to explore further those abstract constructs in the context of visual communication presented itself as I was able to experiment with a newly purchased Apple MacPro 3.1 dual processor equipped with the Adobe CS3 suite. [7] Now I could revisit the Zenon paradox that has captivated so many college years' discussion, use this dynamic environment to access the rigorous tools of the mathematical discipline and illustrate visually interesting occurrences to combine the results with previous researches I had developed in the field of optical illusions (<http://hermay.org/jconstant/oi/oi.htm>).

Maybe not a coincidence, at the particular time (mid 1800 - mid 1900) mathematicians were mapping the various theories upon which this experiment is based, respected authorities in other disciplines, in particular psychology and physiological sciences, were investigating optical aberration. Ewald Hering, Ludimar Hermann produced templates that had an uncanny visual likeness to the work of Cantor and Sierpinski. [3] The turn of the 20st century was also a fertile ground in the deconstructing of two dimensional representation as it was know then with the arrival of cubism, constructivism and the many movements that lead to a radical reinterpretation of the esthetic and visual communication environment.

And last, the focus of this investigation being primarily on two dimensional art and visual communication effectiveness, I will refer the reader looking for scientific rigor to more qualified experts in the field of Laplacian mathematics such as Pelander [1], Strichartz [2] and their work concerning the solvability of differential equations on open subsets of the Sierpinski gasket.

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I – THE MATHEMATICAL GRID

The Sierpinski square fractal pattern requires to start with a solid square and divide it into 9 smaller congruent squares. The original square is scaled 8 times by a factor of $1/3$.

All being proportional - my first challenge was to delimit the canvas in which I could develop my project.

Cartesian coordinates and topological dimensions defined by Sierpinski's contemporaries Lebesgue (1903) and Hausdorff (1918) set the original measurement (point in space) as zero. This was not a viable information I could start from to build a two dimensional representation.

I assigned arbitrarily to my master canvas, an original value of 100 inches.

As I started sketching the first series of subsets I realized very quickly that it would be neither practical nor realistic to pursue further in that direction. (The final square size would have been 0.01524157902758726 inches).

I switched my standard of measurement to the metric system and reverted the progression process, starting with a square figure of 1 millimeter that I would expand 8 times.

The total canvas size would be 2,187 mm or 2.187 m - which should have been - or so I believed - a more manageable size in practical terms.

I used the program Adobe Illustrator, version 13.0.2. to redraw the Sierpinski grid pattern. This vector based graphics application uses mathematical formulae that would insure the best image quality available within my working environment.

I set my initial square – size 1 mm, stroke size 0 - and created a grid 80% black down to 20% in increments of 10%, to fill the progression accordingly, starting the 1 mm square with 80% black down to 20% at the final step (fig. 01).

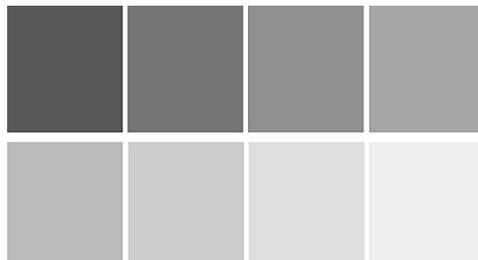


Fig. 01

For good measure I also developed a mirror template to revert the fill process and monitor the inverted optical effect (fig. 02).

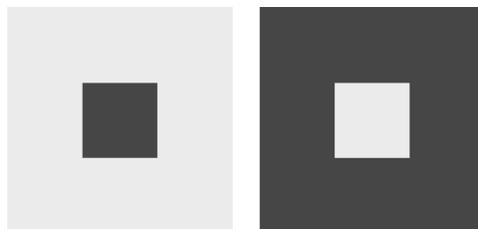


Fig. 02

Plates 1 (mm), - 3 (mm), - 9 (mm), - 27 (mm), - 81 (mm), were completed without problems.

Moving to plate 243 (mm) (fig. 03), I had to start correcting slight changes of x y w h positioning and dimension of the smaller squares. 600.0007 mm turned into in a simple copy-paste step, even after locking all elements on plate 81 (mm).

The computational speed started to slow down considerably during rendering, as I was reformatting and repositioning the elements.

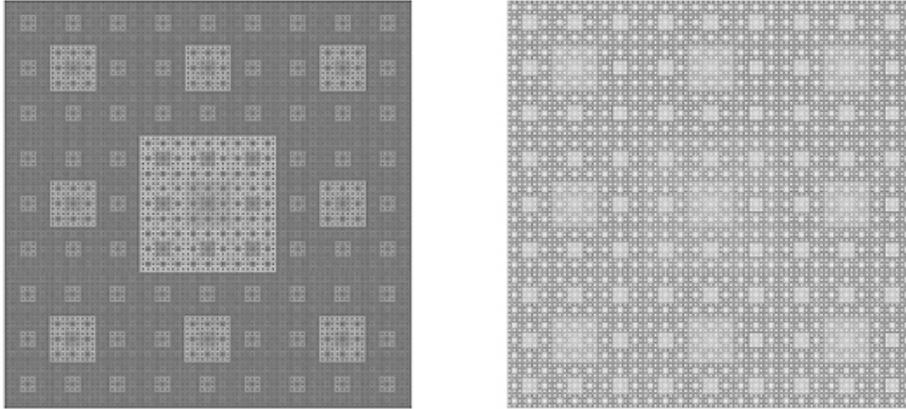


Fig. 03

I also noted that the center square of plate 243 (b) filled with 40% black disappeared in the background because of the hue interference of the squares 1- 3 and 9 (fig. 04).

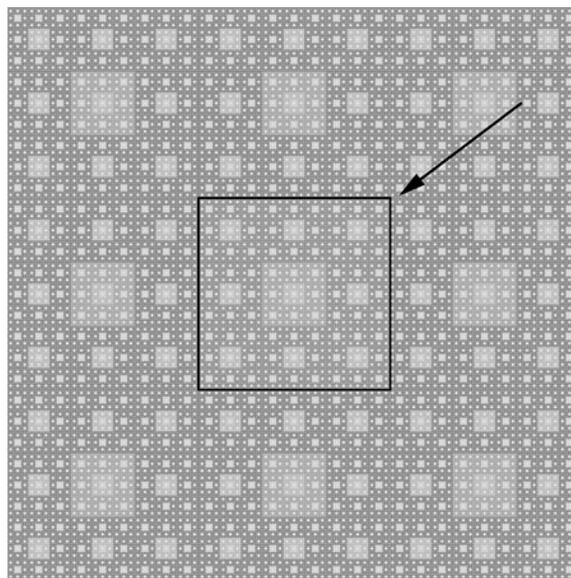


Fig. 04

The computer operating system gave up when I went from step 729 mm to 2187 mm.

Figure (05) is a brief selection of the numerous unexpected pop up windows that came on the screen as I tried every possible combination to either reduce, save, export, convert/rasterize as TIFF, JPG, PNG or PDF.

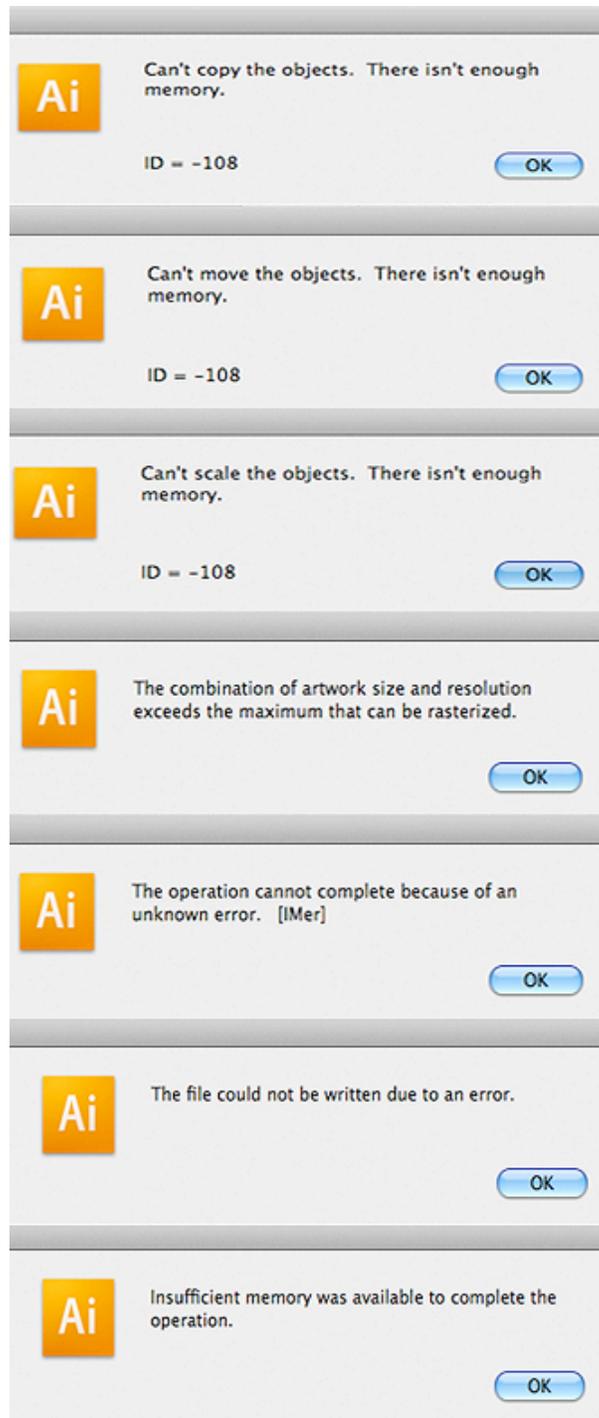


Fig. 05

Another interesting occurrence happened during some – save - actions: a spontaneous degradation of the image.

Following is a file preview screenshot at saving time: most smaller existing squares have disappeared. Closing the document window without saving preserves the original image, whereas clicking OK would save the document as shown in figure (6) below.

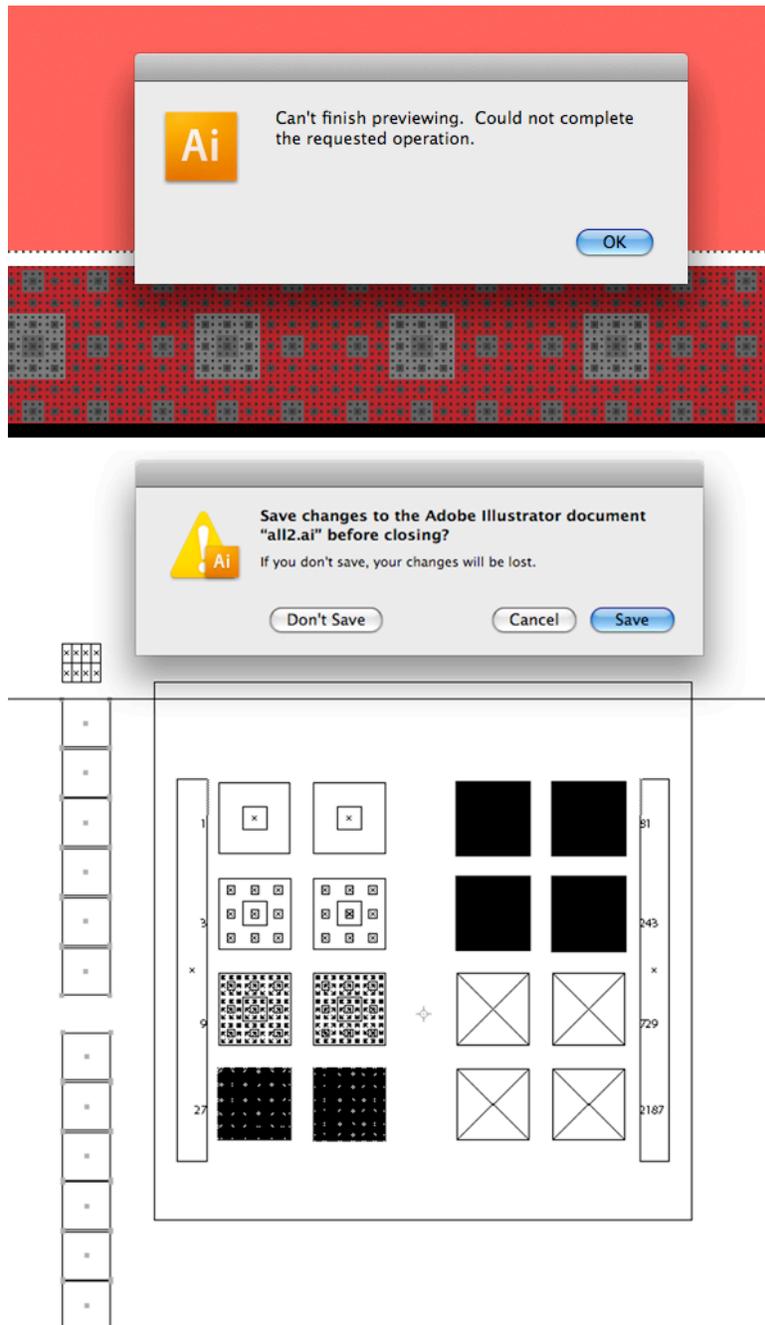


Fig. 06

I kept the computer's Activity Monitor open on a second monitor at the time of the operating system collapse to track any anomaly or overload of the CPU, system memory, disk activity & usage.

The screenshot (fig. 07) shows that the issue was not created by the size of the file or the lack of memory but more likely by the finite limitation of the machine and software processing power.

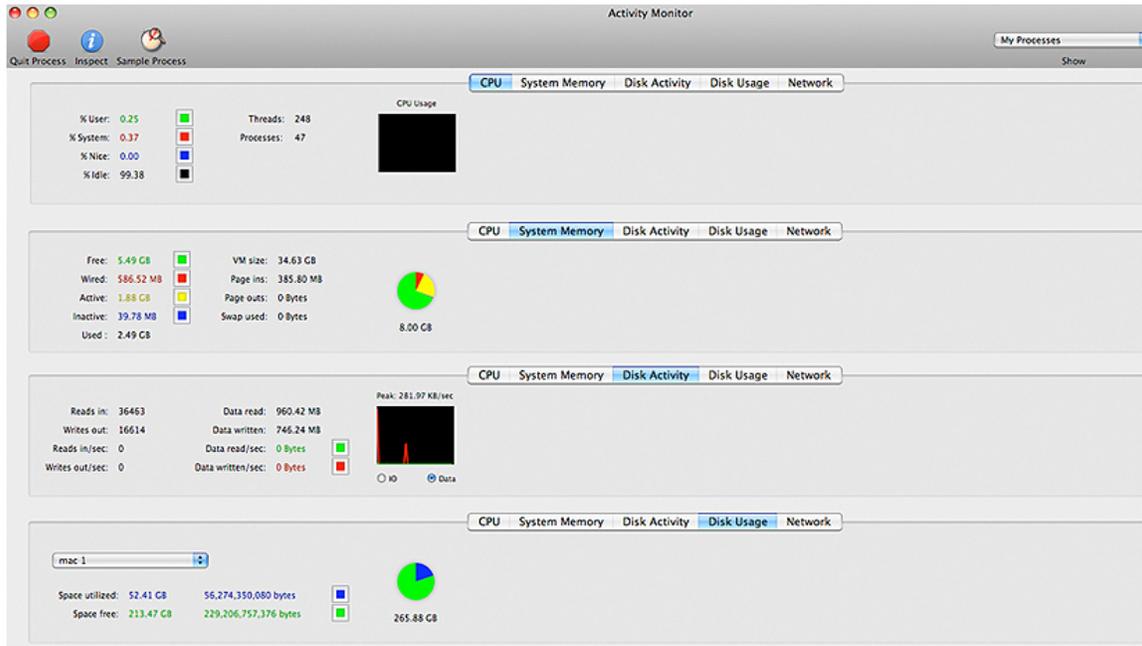


Fig. 07

A quick evaluation of the numbers of squares on each progression plate shows the extend - and limit - of the MacPro computational power in a simple and controlled environment:

Total template size $2,187 \times 2,187 = 4,782,969$ mm.

Total number of squares in template $9^8 = 43,046,721$.

To be consistent with the premises of the project and bring the progression to 2,187, I opted for the only alternative left: a reverse elimination of layers 1, 3, and 9 mm. Working at the edge of the computer processing limit, I proceeded to the step by step removal of the more complex computations and was able to preserve the 1-3-9 mm in the larger squares (fig. 08).

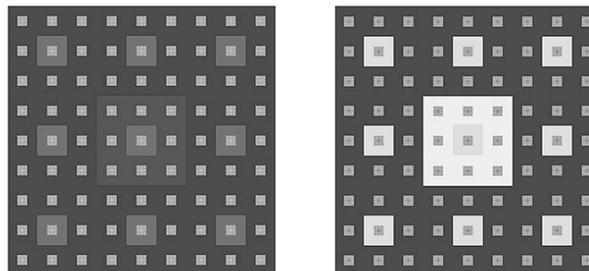


Fig. 08

The individual squares were then regrouped and repositioned on a solid red background to highlight the fractal progression and grey to white, white to grey optical effect (fig. 09).

*Variation on the Sierpinsky - cubic regression
Generalization of the Cantor set in two dimensions*

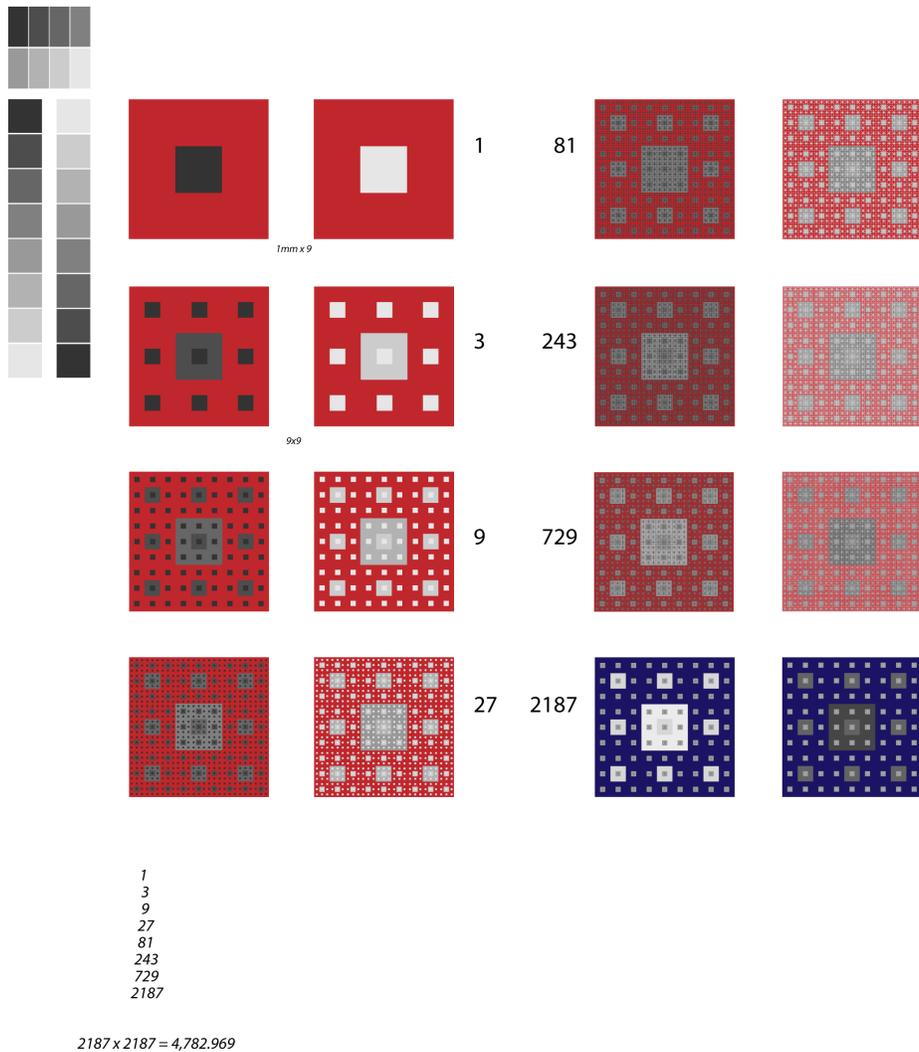


Fig. 09

II – OPTICAL ILLUSION

Can we really trust what our eyes see? [3][4][5]. Now I could move to the second stage of the project and integrate the Sierpinsky templates in selected optical aberrations blueprints .

I transferred the templates 2187 into Photoshop 10.00 and worked in a new canvas, extracting or blending selected elements of existing optical illusion grids as needed.

Below is a brief descriptive of the process.

- **Hermann grid:**

Dark patches appear in the street crossings, except the ones that you are directly looking at.

If you look around in the neighboring figure you will notice the appearance and disappearance of black dots at the crossings. (*Invoked to explain Florida's election problems in 2001: "Count the black dots, recount to confirm..."*) (fig. 10).

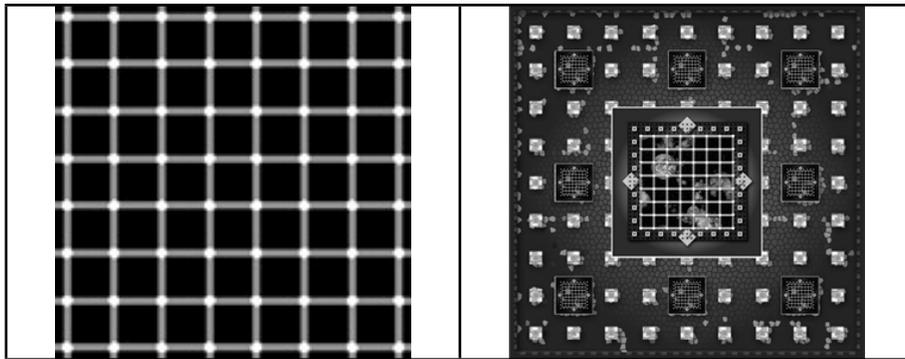


Fig. 10

- **The Hering illusion:**

Discovered in the 19th century by German physiologist Ewald Hering.

Two straight lines appear curved or bowed in the context of intersecting lines with orientations that change progressively. Hiding the oblique lines from the view will reveal the fact that the horizontal bars are equally straight (fig. 11).

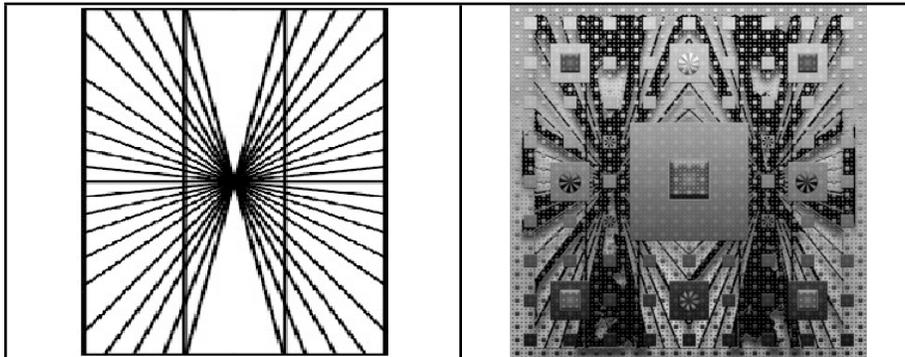


Fig. 11

- **The Aitken wheel:**

In homage to Robert Grant Aitken astronomer, director of the Lick observatory and president of the University of California. Concentrating on the wheel center for a few minutes will make the wheel spin (fig. 13).

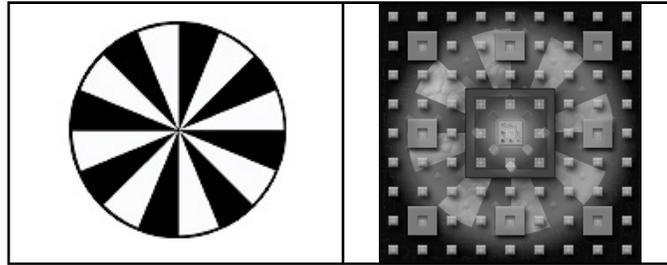


Fig. 13

- **The Ebbinghaus - Titchener illusion:**

Identical objects are perceived as unequal in size when they are surrounded by objects of a different size. This kind of assimilation illusion is also associated with the name of Franz Joseph Delboeuf, a Belgian philosopher. The copies of the same shape - circles and rings in our case - appear to inherit properties of their environment. Two equal circles - one inside a bigger circle, the other containing a smaller circle - seem to have different sizes (fig. 14).

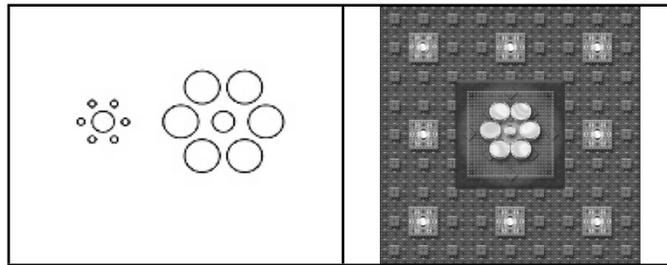


Fig. 14

- **The Wundt illusion:**

Considered the founder of experimental psychology, Wilhelm Wundt introduced cognitive principles to the psychology community in the late 1880s. He was the first one to report on the phenomenon. The overestimation of the subtense of acute angles and the underestimation of obtuse ones was first reported first by Wundt (fig. 12).

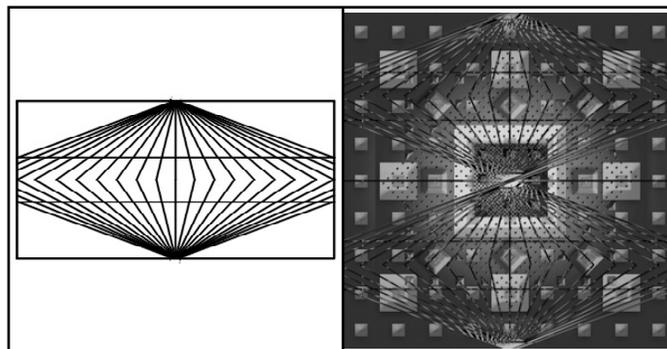


Fig. 12

III - COLOR

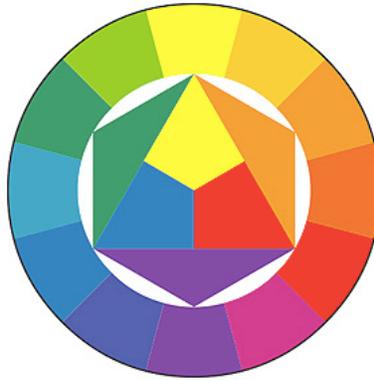


Fig. 15

In the early 1900's, Johannes Itten, color instructor at the Bauhaus and trained in psychoanalytic theory developed a color wheel that took into consideration the subjective feeling often associated with objective color, and the psychic and emotional values of colors [6].

I used Itten's color wheel (figure 15) as a reference in many previous works. I thought it would be fitting to expand my project to include another layer of manipulation inspired by the work of great figure of the early 1900. However I was also aware that creating the original object in a digital environment, I would need to apply some level of color correction to the final printed result to keep the color scheme consistent.

This challenge was made considerably easier by a fairly recent tool made available by the Adobe Lab, the Kuler application [7] that allows the user to create and color-test various combinations of hues in real time (fig. 16).

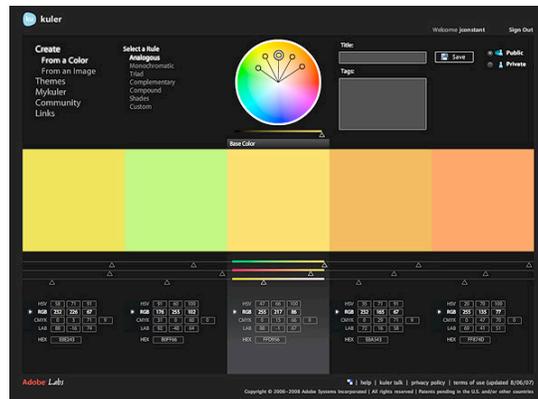


Fig. 16

Working in separate layers, I blended several color schemes alternatives I found most effective in bringing up the relation between shape, size and positioning of the objects.

Below is an example of a progression from black & white to a set color scheme using a digitally enhanced Itten color wheel.



Fig. 15

I completed 21 original plates, resolution 300 pixel per inch, dimension 15x15 inches. and posted the results on the internet gallery at: <http://hermay.org/jconstant/cantor> .

I also controlled the stability and consistency of the design between the backlighted (RGB) image and front-lighted (CMYK) plate by printing each plate on an professional quality Inkjet Epson 7600 printer. The final outcomes are available size 22x22 on fine art, archival paper and demonstrate further the dynamic relevance of the technical elements that motivated the demonstration.



CONCLUSION

As it happens often in an exploratory journey, I found challenges I was not expecting and got rewards I was not planning on finding. I developed a deeper respect for the mathematic and scientific methodology and the minds that map out the objective understanding of the environment. I also grew respect and appreciation for the engineers and graphic programmers that allow us to expand the understanding of abstract concept in meaningful visualizations.

The coherence of the scientific and artistic discourse at the turn of the century still stands today as a solid technical and intellectual proposition all visual communication endeavors can benefit from:

- The tools of mathematics, psychological and optical physiology enhance visual propositions in all forms of expression.
- Good communication gain from tapping scientific data to expand its effectiveness.
- Our computers' operating systems may be still finite and imperfect today but the on-going collaboration of trained specialists in various disciplines will continue to advance further our common knowledge and provide new wealth of opportunities for all.



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This project could not have been completed without their help and support.

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- [4] Edward H. Adelson. *Lightness Perception and Lightness Illusions*. Department of Brain and Cognitive Sciences. Massachusetts Institute of Technology.
- [5] Thomas D. Albright, Maarten J. van der Smagt. *Motion after-effect direction depends on depth ordering in the test pattern* Vision Center Laboratory & Howard Hughes Medical Institute, The Salk Institute for Biological Studies.
- [6] Johannes Itten. *The Elements of color*.
- [7] Computer profile:

Hardware:

- Model Name: Mac Pro
- Model Identifier: MacPro3,1
- Processor Name: Quad-Core Intel Xeon
- Processor Speed: 3.2 GHz
- Number Of Processors: 2
- Total Number Of Cores: 8
- L2 Cache (per processor): 12 MB
- Memory: 8 GB
- Bus Speed: 1.6 GHz
- Boot ROM Version: MP31.006C.B05
- SMC Version: 1.25f4
- Graphic & Display
 - NVIDIA GeForce 8800 GT
 - VRAM (Total): 512 MB

Software:

- Mac OSX v.10.5.4
- Adobe CS3 MC. Photoshop Illustrator Bridge
- AdobeLab Kuler API (Application Programming Interface)